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(ii) upper sideband ($\omega_c + \omega_m$) having amplitude $\frac{m_a : A}{2}$

(iii) Lower sideband ($\omega_c - \omega_m$) having amplitude $\frac{m_a \cdot A}{2}$

3. Frequency Spectrum

With the help of these frequency components, we can plot the frequency-spectrum of single-tone amplitude modulated (AM) wave. Figure 2.4 (a) shows the one-sided frequency spectrum of single-tone AM wave.

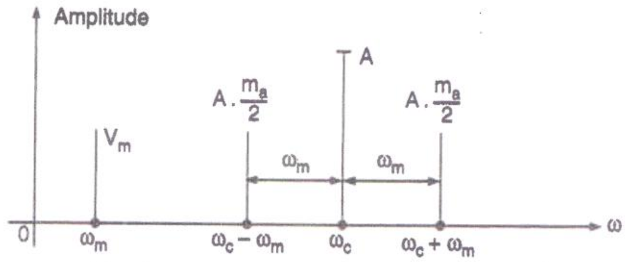

Fig. 2.4 (a) Single-sided frequency spectrum of single-tone AM wave

EXAMPLE 2.1. The tuned-circuit of the oscillator in an AM transmitter uses a 50 μH coil and a 1 nF capacitor. Now, if the oscillator output is modulated by audio frequencies upto 8 kHz, then find the frequency range occupied by the sidebands.

Solution : The oscillator in AM transmitter is used to generate high carrier frequency. Hence, the resonance frequency of the oscillator will be the carrier frequency.

Therefore,

Carrier frequency, $f_c = \frac{1}{2\pi\sqrt{LC}}$

Here given that $L = 50\ \mu H$

$L = 50 \times 10^{-6}$ H

and $C = 1$ nF $= 1 \times 10^{-9}$ F

Thus,

$$f_c = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 1 \times 10^{-9}}} = \frac{1}{2\pi\sqrt{5 \times 10^{-14}}}$$

$$f_c = \frac{1}{2\pi \times 10^{-7} \times \sqrt{5}} = 7.12 \times 10^5 \text{ Hz} = 712 \text{ kHz}$$

Now, it is given that the highest modulating frequency is 8 kHz.

Important Point: *Therefore, the frequency range occupied by the sidebands will range from 8 kHz above to 8 kHz below the carrier frequency, extending from 712 kHz to 720 kHz.* **Ans.**

EXAMPLE 2.2. A modulating signal 10 sin($2\pi \times 10^3$t) is used to modulate a carrier signal 20 sin ($2\pi \times 10^4$t). Determine the modulation index, percentage modulation, frequencies of the sideband components and their amplitudes. What will be the bandwidth of the modulated signal?

Solution: (i) The modulating signal $v_m = 10 \sin(2\pi \times 10^3 t)$.

Let us compare this with the following expression

$$v_m = V_m \sin(2\pi f_m t)$$

Then, we get $V_m = 10$ volt, $f_m = 1 \times 10^3$ Hz $= 1$ kHz

(ii) The carrier signal $v_c = 20 \sin(2\pi \times 10^4 t)$

Comparing this with the expression $v_c = V_c \sin(2\pi f_c t)$, we obtain

$$V_c = 20 \text{ volt}, f_c = 1 \times 10^4 \text{ Hz} = 10 \text{ kHz}$$

(a) Modulation index and percentage modulation:

We know that $m = \frac{V_m}{V_c} = \frac{10}{20} = 0.5$ and % modulation $= 0.5 \times 100 = 50$ %

(b) Frequencies of sideband components:

(i) Upper sideband $f_{USB} = f_c + f_m = (10 + 1) = 11$ kHz

(ii) Lower sideband $f_{LSB} = f_c - f_m = (10 - 1) = 9$ kHz

(c) Amplitudes of sidebands:

The amplitudes of upper as well as the lower sideband are given by,

$$\text{Amplitude of each sideband} = \frac{mV_c}{2} = \frac{0.5 \times 20}{2} = 5 \text{ Volt}$$

(d) Bandwidth $= 2\,f_m = 2 \times 1 \text{ kHz} = 2 \text{ kHz}$ **Ans.**

2.6. POWER CONTENT IN AM WAVE

(U.P. Tech, Sem. Exam., 2004-05)

1. Definition

It may be observed from the expression of AM wave that the carrier component of the amplitude modulated wave has the same amplitude as unmodulated carrier. In addition to carrier component, the modulated wave consists of two sideband components. It means that the modulated wave contains more power than the unmodulated carrier. However, since the amplitudes of two sidebands depend upon the modulation index, it may be anticipated that the total power of the amplitude modulated wave would depend upon the modulation index also. In this section, we shall find the power contents of the carrier and the sidebands.

We know that the general expression of AM wave is given as

$$s(t) = A \cos \omega_c t + x(t) \cos \omega_c t \qquad ...(2.25)$$

The total power P of the AM wave is the sum of the carrier power P_c and sideband power P_s.

2. Carrier Power

The carrier power P_c is equal to the mean-square (ms) value of the carrier term $A \cos\omega_c t$ i.e.

$$P_c = \text{mean square value of } A \cos\omega_c t$$

$$P_c = [A \cos\omega_c t]^2 = \frac{1}{2\pi}\int_0^{2\pi} A^2 \cos^2 \omega_c t.\, dt^* = \frac{A^2}{2} \qquad ...(2.26)$$

3. Sideband Power

The sideband power P_s is equal to the mean square value of the sideband term $x(t) \cos\omega_c t$, i.e.

$$P_s = \text{mean square value of } x(t) \cos\omega_c t$$

$$P_s = [x(t) \cos\omega_c t]^2 = \frac{1}{2\pi}\int_0^{2\pi} x^2(t) \cos^2 \omega_c t\, dt$$

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\left[2\cos^2 \omega_c t\right] x^2(t)\, dt$$

or

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} x^2(t)\, dt + \frac{1}{2\pi}\int_0^{2\pi} x^2(t) \cos 2\omega_c t\, dt \qquad ...(2.27)$$

In AM generation, a Band pass filter (BPF) or a tuned circuit tuned to carrier frequency ω_c is used to filter out the second integral term.

Therefore,

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \left[\frac{1}{2} x^2(t)\right] dt$$

or

$$P_s = \text{mean square (ms) value of } \frac{1}{2} x^2(t) = \frac{1}{2}\overline{x^2(t)} \qquad ...(2.28)$$

However, the total sideband power P_s is due to the equal contributions of the upper and lower sidebands. Hence, the power carried by the upper and the lower sidebands will be

---

* since period of the signal $A\cos \omega_c t$ is 2π.

(179)

The sideband power $P_s = \frac{1}{2}\overline{x^2(t)} = \frac{1}{2}\overline{(V_m \cos\omega_m t)^2}$

$$P_s = \frac{1}{2}\frac{V_m{}^2}{2} = \frac{1}{4}V_m{}^2 \qquad ...(2.34)$$

We know that the total modulated power P_t is the sum of P_c and P_s.

Therefore $P_t = P_c + P_s = \frac{A^2}{2} + \frac{1}{4}\cdot V_{m^2} = \frac{A^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{A}\right)^2\right]$

But $\frac{V_m}{A} = \frac{\text{Maximum baseband amplitude}}{\text{Maximum carrier amplitude}} = m_a$ = modulation index for AM

Hence $P_t = \frac{A^2}{2}\left[1 + \frac{1}{2}\cdot m_a{}^2\right]$

But $\frac{A^2}{2} = P_c$ = carrier power

Therefore, $P_t = P_c\left(1 + \frac{m_a{}^2}{2}\right)$ ...(2.35)

EXAMPLE 2.3. A 400 watts carrier is modulated to a depth of 75 percent. Find the total power in the amplitude-modulated wave. Assume the modulating signal to be a sinusoidal one.

Solution : We know that for a sinusoidal modulating signal, the total power is expressed as

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$$

where P_t = total power or modulated power
P_c = carrier power or unmodulated power
m_a = modulation index

Given that, P_c = 400 watts
m_a = 75 percent = 0.75

Therefore, $P_t = P_c\left(1 + \frac{m_a^2}{2}\right) = 400\left(1 + \frac{0.75^2}{2}\right) = 512.5$ watts **Ans.**

EXAMPLE 2.4. An AM broadcast radio transfer radiates 10 K watts of power if modulation percentage is 60. Calculate how much of this is the carrier power.

Solution : We know that the total power is expressed as

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \qquad ...(i)$$

where P_t = total power or modulated power
P_c = carrier power or unmodulated power
m_a = modulation index

Given that, P_t = 10 K watts
m_a = 60 percent = 0.6

From equation (i), we get

$$P_c = \frac{P_t}{1 + \frac{m_a^2}{2}} = \frac{10}{1 + \frac{0.6^2}{2}} = \frac{10}{1.18} = 8.47 \text{ kW}$$ **Ans.**

2.9 CURRENT CALCULATION FOR SINGLE-TONE AM

1. Definition

In AM, it is generally more convenient to measure the AM transmitter current than the power. In this case, the modulation index may be calculated from the values of unmodulated and modulated currents in the AM transmitter.

2. Mathematical Expression

Let I_c be the r.m.s. value of the carrier or unmodulated current and I_t be the r.m.s. value of the total or modulated current of an AM transmitter. Let R be the antenna resistance through which these currents flow.

Now, we know that for a single-tone modulation the power relation is expressed as

$$P_t = P_c\left(1+\frac{m_a^2}{2}\right) \quad \text{...(2.36)}$$

where P_t = total or modulated power

P_c = carrier or unmodulated power

m_a = modulation index

From equation (2.36), we may write

$$\frac{P_t}{P_c} = 1+\frac{m_a^2}{2}$$

or
$$\frac{I_t^2 \cdot R}{I_c^2 \cdot R} = 1+\frac{m_a^2}{2} \quad \text{or} \quad \frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}}$$

or
$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}} \quad \text{...(2.37)}$$

EXAMPLE 2.5. The antenna current of an AM transmitter is 8 A if only the carrier is sent, but it increases to 8.93 A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also find the antenna current if the percent of modulation changes to 0.8.

(U.P. Tech, Sem., Exam., 2004-05)

Solution : (i) The current relation for a single-tone amplitude modulation is expressed as

$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}} \quad \text{(i)}$$

where I_t = total or modulated current

I_c = carrier or unmodulated current

m_a = modulation index

Using equation (i), we get

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}}$$

or
$$\left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m_a^2}{2} \quad \text{or} \quad \frac{m_a^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1$$

or
$$m_a^2 = 2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right] \quad \text{or} \quad m_a = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]}$$

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Putting all the given values, we have

$$m_a = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]} = \sqrt{2\left[(1.116)^2 - 1\right]} = \sqrt{2(1.246-1)} = \sqrt{0.492}$$

$$m_a = 0.701 = 70.1\%$$

(ii) Since $I_t = I_c\sqrt{1 + \frac{m_a^2}{2}}$

Here, $I_c = 8A$

and $m_a = 0.8$

Therefore, $I_t = 8 \times \sqrt{1 + \frac{0.8^2}{2}} = 8\sqrt{1 + \frac{0.64}{2}} = 8\sqrt{1.32} = 8 \times 1.149 = 9.19$ A **Ans.**

EXAMPLE 2.6. The antenna current of an AM transmitter is 10 ampere when it is modulated to depth of 30% by an audio signal. It increases to 11 ampere when another signal modulates the carrier signal. What will be the modulation index due to second signal?

Solution: It is given that $I_{t1} = 10$ amp, $m_1 = 0.3$, $I_{t2} = 11$ amp

$$\left[\frac{I_{t1}}{I_c}\right]^2 = 1 + \frac{m_1^2}{2}$$

Therefore, $$I_c = \frac{I_{t1}}{\left[1 + \frac{m_1^2}{2}\right]^{1/2}} = \frac{10}{\left[1 + \frac{(0.3)^2}{2}\right]^{1/2}} = 9.78 \text{ Amp} \qquad \text{...(i)}$$

(ii) After modulating with the second signal, we have

$$I_{t2}^2 = I_c^2\left[1 + \frac{m_t^2}{2}\right]$$

or $$m_t = \sqrt{2\left[\left(\frac{I_{t2}}{I_c}\right)^2 - 1\right]} = \sqrt{2\left[\left(\frac{11}{9.78}\right)^2 - 1\right]}$$

Therefore, $m_t = 0.73$ ...(ii)

(iii) But, we have $m_t = \left[m_1^2 + m_2^2\right]^{1/2}$

or $m_2 = \left[m_t^2 - m_1^2\right]^{1/2} = [(0.73)^2 - (0.3)^2]^{1/2} = 0.66$ or 66% **Ans.** ...(iii)

EXAMPLE 2.7. A carrier wave is represented by the expression $v_c(t) = 10 \sin \omega t$. Draw the waveform of an AM wave for m = 0.5.

Solution: Given that $v_c = 10 \sin \omega t$

Therefore, $V_c = 10$ Volts

First, let us calculate V_m from V_c.

Since $m = \frac{V_m}{V_c}$

$V_m = m \times V_c = 0.5 \times 10$

$= 5$ Volt ...(1)

Hence, $V_{max} = V_c + V_m = 10 + 5 = 15$ Volt

and $V_{min} = V_c - V_m = 10 - 5 = 5$ Volt

Now, the AM wave for m = 0.5 will be shown in figure 2.5.

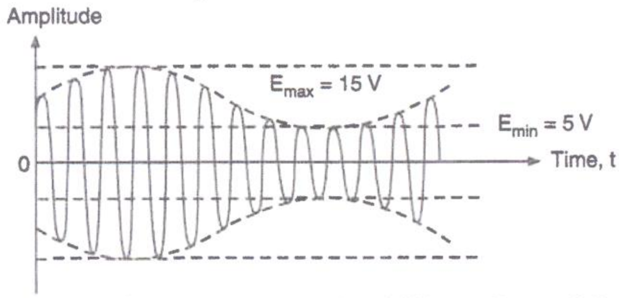

Fig. 2.5. Graphical representation of AM wave for m = 0.5

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EXAMPLE 2.8. Prove that in amplitude modulation, maximum average power transmitted by an antenna is 1.5 times the carrier power.

Solution: Let P_c = Unmodulated carrier power

P_t = Transmitted AM power

m = Modulation index

We know that
$$P_t = P_c\left[1+\frac{m^2}{2}\right] \qquad \text{...(i)}$$

The maximum value of m without introducing distortion in the modulated wave is m = 1. Substituting this value in equation (i), we obtain

$$P_t(\text{max}) = \left[1+\frac{1}{2}\right] = 1.5\ P_c$$ **Hence Proved**

EXAMPLE 2.9. The carrier amplitude after AM varies between 4 volts and 1 volt. Calculate depth of modulation.

Solution: We know that we can calculate the value of modulation index with the help of the following expression:

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

Substituting V_{max} = 4 Volt and V_{min} = 1 Volt, we get,

$$m = \frac{4-1}{4+1} = \frac{3}{5} = 0.6 \text{ or } 60\%. \quad \textbf{Ans.}$$

EXAMPLE 2.10. The antenna current of AM broadcast transmitter modulated to the depth of 40% by an audio sine wave is 11 Amp. It increases to 12 Amp. as a result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?

Solution: Given that m_1 = 0.4, I_{t1} = 11 Amp., I_{t2} = 12 Amp.

We know that
$$\left[\frac{I_{t1}}{I_c}\right]^2 = 1 + \frac{m_1^2}{2}$$

Simplifying, we get
$$I_c = \frac{I_{t1}}{\left[1+\frac{m_1^2}{2}\right]^{1/2}} = \frac{11}{\left[1+\frac{(0.4)^2}{2}\right]^{1/2}} = 10.58 \text{ Amp.} \qquad \text{...(i)}$$

After modulation with the second signal, we have

$$\left[\frac{I_{t2}}{I_c}\right]^2 = 1 + \frac{m_t^2}{2}$$

where m_t is the total modulation index.

Therefore, we have
$$I_{t2}^2 = I_c^2\left[1 + \frac{m_t^2}{2}\right]$$

or
$$m_t = \left[2\left\{\frac{I_{t2}^2}{I_c^2} - 1\right\}\right]^{1/2}$$

Substituting all the values, we obtain

$$m_t = \left[2\left\{\left(\frac{12}{10.58}\right)^2 - 1\right\}\right]^{1/2} = 0.7553 \qquad \text{...(ii)}$$

Total modulation index $m_t = \left[m_1^2 + m_2^2\right]^{1/2}$

or $m_2 = [m_t^2 - m_1^2]^{1/2} = [(0.7553)^2 - (0.4)^2]^{1/2}$

or $m_2 = 0.6407$ or 64.07% **Ans.**

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EXAMPLE 2.11. A certain transmitter radiates 10 kW with carrier unmodulated and 12 kW when the carrier is sinusoidally modulated. Calculate the modulation index. If another sinewave corresponding to 50% modulation is transmitted simultaneously, determine the total radiated power.

Solution: Given that $P_c = 10$ kW, $P_t = 12$ kW, $m_2 = 0.5$

The modulation index m_1 is given by,

$$\frac{P_t}{P_c} = 1 + \frac{m_1^2}{2}$$

or

$$\frac{12}{10} = \left[1 + \frac{m_1^2}{2}\right]$$

or

$$m_1 = 0.6324 \qquad \text{...(i)}$$

The total modulation index m_t is given by

$$m_t = [m_1^2 + m_2^2]^{1/2} = [(0.6324)^2 + (0.5)^2]^{1/2} = 0.8 \qquad \text{...(ii)}$$

The new transmitted power is given by

$$P_t = P_c\left[1 + \frac{m_t^2}{2}\right] = 10\,\text{kW}\left[1 + \frac{(0.8)^2}{2}\right] = 13.2 \text{ kW.} \quad \textbf{Ans.}$$